$$\cancel{(x_1 + x_2 e_2) + (x_3 + x_4 e_2)}$$

$$q = (x_4 + x_3 e_3) + (x_2 + x_1 e_3) e_2$$

$$d\bar{q} = (x_4 - x_3 e_3) - (x_2 - x_1 e_3) e_2$$

$$= \sin\theta \cos\alpha \left(\underbrace{\cos(\phi_1 + \phi_2) - e_3 \sin(\phi_1 + \phi_2)}_{\Downarrow \; e^{-i e_3(\phi_1 + \phi_2)}} \right)$$

$$= \cancel{1 - i e_3}$$

$$= \underline{\sin\theta \cdot \cos\alpha \, e^{-i e_3(\phi_1 + \phi_2)}} \; \cancel{+ \sin \, e_2}$$

$$- \underline{e_2 \sin\theta \sin\alpha \, e^{-i e_3(\phi_1 + \phi_2)}}$$

$$\underline{\cos\theta \cos\alpha \, e^{-i e_3(\phi_1 - \phi_2)} - \cos\theta \, e_2 \sin\alpha \, e^{-i(\phi_1 - \phi_2})}$$

$$\sin\theta \left[\cos\alpha \, e^{i e_3(\phi_1 - \phi_2)} + \overset{e_2}{\sin\alpha} \, e^{i e_3(\phi_1 + \phi_2)} \right]$$

$$\cdot \cos\theta \left[\cos\alpha \, e^{-i e_3(\phi_1 - \phi_2)} - \sin\alpha \, e^{i e_3(\phi_1 + \phi_1)} \right]$$

$$= \sin\theta \cdot \cos\theta \cdot d\theta + \sin\theta \cdot \cos\theta \cdot \sin\alpha \cos\alpha \left[e^{i e_3 2\phi_2} - e^{-i e_3 \phi_2} \right]$$

$$\bar{q} = (x_4 - x_3 e_3) \cancel{e_2}\,(x_2 - e_3 x_1) e_2$$

$$q = (x_4 + x_3 e_3) + e_2 (x_2 + e_3 x_1)$$

$$d\bar{q} = \sin\theta \Big[\cos\alpha \big(\cos(\phi_1 - \phi_2) - \sin(\phi_1 - \phi_2) e_3 \big)$$

$$- \sin\alpha \big(\cos(\phi_1 + \phi_2) - \sin(\phi_1 + \phi_2) e_3 \big) e_2 \Big)$$

$$= \sin\theta \Big(\cos\alpha\, e^{-e_3(\phi_1 - \phi_2)} - \sin\alpha\, e^{-e_3(\phi_1 + \phi_2)} e_2 \Big)$$

$$\cos\theta \cdot \sin\theta \Big(1 + \sin\alpha\cos\alpha \big(e_2\, e^{i e_3(\phi_1 + \phi_2) - e_3(\phi_1 - \phi_2)}$$

$$- e^{i e_3(\phi_1 - \phi_2) - e_3(\phi_1 + \phi_2)} e_2 \big) \Big)$$

$$= \sin\theta \cdot \cos\theta \Big(1 + \sin\alpha\cos\alpha \big(e_2 e^{i 2 e_3 \phi_2} - e^{-i 2 e_3 \phi_2} e_2 \big) \Big)$$

de

$$-\sin\alpha + \cos\alpha$$

$$\Big(\cos\alpha\, e^{i e_3(\phi_1 - \phi_2)} + e_2 \sin\alpha\, e^{i e_3(\phi_1 + \phi_2)} \Big) \Big(- \sin\alpha\, e^{-i e_3(\phi_1 - \phi_2)}$$

$$- \cos\alpha\, e^{i(e_3(\phi_1 + \phi_2))} e_2 \Big)$$

$$\cancel{(x_1 + x_2 e_2) + (x_3 + x_4 e_3)}$$

$$q = (x_4 + x_3 e_3) + (x_2 + x_1 e_3) e_2$$

$$d\bar{q} = (x_4 - x_3 e_3) - (x_2 - x_1 e_3) e_2$$

$$= \sin\theta \cos\alpha \left(\underline{\cos(\phi_1+\phi_2) - e_3 \sin(\phi_1+\phi_2)} \right)$$

$$\Downarrow$$

$$e^{-ie_3(\phi_1+\phi_2)}$$

$$= \cancel{1 - ie_3}$$

$$= \underline{\sin\theta \cdot \cos\alpha \, e^{-ie_3(\phi_1+\phi_2)}} \cancel{+ \sin \, e_2}$$

$$- \underline{e_2 \sin\theta \sin\alpha \, e^{-ie_3(\phi_1+\phi_2)}}$$

$$\underline{\cos\theta \cos\alpha \, e^{-ie_3(\phi_1-\phi_2)} - \cos\theta \, e_2 \sin\alpha \, e^{-i(\phi_1-\phi_2}})$$

$$\sin\theta \left[\cos\alpha \, e^{ie_3(\phi_1-\phi_2)} + \sin\alpha \, e^{ie_3(\phi_1+\phi_2)} e_2 \right]$$

$$\cdot \cos\theta \left[\cos\alpha \, e^{-ie_3(\phi_1-\phi_2)} - \sin\alpha \, e^{ie_3(\phi_1+\phi_1)} \right]$$

$$= \sin\theta \cdot \cos\theta \cdot d\theta + \sin\theta \cdot \cos\theta \cdot \sin\alpha \cos\alpha \left[e^{ie_3 2\phi_2} - e^{-ie_3\phi_2} \right]$$

$$\bar{q} = (x_4 - x_3 e_3) - e_2 (x_2 - e_3 x_1) e_2$$

$$q = (x_4 + x_3 e_3) + e_2 (x_2 + e_3 x_1)$$

$$d\bar{q} = \sin\theta \Big[\cos\alpha \big(\cos(\phi_1 - \phi_2) - \sin(\phi_1 - \phi_2) e_3 \big)$$

$$- \sin\alpha \big(\cos(\phi_1 + \phi_2) - \sin(\phi_1 + \phi_2) e_3 \big) e_2 \Big)$$

$$= \sin\theta \left(\cos\alpha\, e^{-e_3(\phi_1 - \phi_2)} - \sin\alpha\, e^{-e_3(\phi_1 + \phi_2)} e_2 \right)$$

$$\cos\theta \cdot \sin\theta \Big(1 + \sin\alpha\cos\alpha \big(e_2\, e^{i e_3(\phi_1 + \phi_2) - e_3(\phi_1 - \phi_2)}$$

$$- e^{i e_3(\phi_1 - \phi_2) - e_3(\phi_1 + \phi_2)} e_2 \big) \Big)$$

$$= \sin\theta \cdot \cos\theta \left(1 + \sin\alpha\cos\alpha \left(e_2 e^{i 2 e_3 \phi_2} - e^{-i 2 e_3 \phi_2} e_2 \right) \right)$$

$d e$

$$- \sin\alpha + \cos\alpha$$

$$\left(\cos\alpha\, e^{i e_3(\phi_1 - \phi_2)} + \sin\alpha\, e^{i e_3(\phi_1 + \phi_2)} e_2 \right) \left(- \sin\alpha\, e^{-i e_3(\phi_1 - \phi_2)} \right.$$

$$\left. - \cos\alpha\, e^{i(e_3(\phi_1 + \phi_2))} e_2 \right)$$

$$dz_1 = \cos\theta \sin\alpha\, e^{i(\phi_1+\phi_2)}\, d\theta + \sin\theta \cdot \cos\alpha\, e^{i(\phi_1+\phi_2)}$$

$$+ i \sin\theta \cdot \sin\alpha\, e^{i(\phi_1+\phi_2)}\, d(\phi_1+\phi_2)$$

$$d\bar{z}_1 = \cos\theta \cdot \sin\alpha\, e^{-i(\phi_1+\phi_2)}\, d\theta + \sin\theta \cdot \cos\alpha\, e^{-i(d_1+\phi_2)}$$

$$- i \sin\theta \sin\alpha\, e^{-i(\phi_1+\phi_2)}\, d(\phi_1 + \phi_2)$$

$$dz_1 d\bar{z}_1 = \cos^2\theta \cdot \sin^2\alpha\, d\theta^2 + \sin^2\theta \cos^2\alpha\, d\alpha^2$$

$$+ \sin^2\theta \sin^2\alpha\, d\phi_1^2$$

$$ds^2 = \underline{d\theta^2 + \sin^2\theta\, d\alpha^2 + \sin^2\theta \sin^2\alpha\, d\phi_1^2}$$

$$\underline{\sin^2\theta\, \cancel{\sin}\, \cos^2\alpha\, d\phi_2}$$

$$H = \frac{1}{2m} \sum_i \left(-i\hbar \frac{\partial}{\partial x_i} + \frac{e}{c} g A_i^a t_a \right)^2$$

$$= \sum \frac{1}{2m} \left\{ \sum_i -\hbar^2 \frac{\partial^2}{\partial x_i^2} + \left(\frac{eg}{c}\right)^2 \sum_{i=1}^{4} A_i^a A_j^b \right.$$

$$t_a t_b + \sum_i -i\hbar \left(\frac{\partial}{\partial x_i} \cdot \frac{eg}{c} A_j^a t_a + \frac{eg}{c} A_i^a t_a \right.$$

$$\left. \frac{\partial}{\partial x_j} \right)$$

term1: 即 laplace operator 仅为

$\Rightarrow$

term2: $\frac{1}{2m}\left(\frac{eg}{c}\right)^2 \sum_{i=1}^{4} \underline{A_i^a A_{j_i}^b} \, t_a \cdot t_b$

$\Downarrow$

$$\underline{\frac{1}{4(1+x_5)^2} \eta^a_{\mu\nu} \eta^b_{\rho\sigma} \, t_a \cdot t_b \, x_\nu x_\sigma}$$

$\Downarrow$

需要重新计算

$$\varepsilon_{abc} \eta^a_{\mu\nu} \eta^b_{\rho\sigma} = \eta_{-}$$

其中 $-i\hbar \frac{\partial}{\partial x_i} \cdot \frac{-eg/c}{2(1+x_5)} \eta^a_{im} x_m e_a$

$= i\hbar \frac{eg}{2c} \frac{1}{(1+x_5)} \eta^a_{im} x_m \frac{\partial}{\partial x_i} e_a$

$= -\frac{eg}{2c} \frac{1}{1+x_5} \vec{L}^a \cdot \vec{e_a}$

$K_a = \frac{1}{2}\eta^a_{\mu\nu} L_{\mu\nu} = \frac{1}{2}\eta_{\mu\nu}(x_\mu p_\nu - x_\nu p_\mu)$

$= \eta^a_{\mu\nu} x_\mu p_\nu$

- configuration $A_\mu = \frac{1}{\sqrt{2}}(\eta^a_{\mu\nu} + \bar{\eta}^a_{\mu\nu}) x_\nu \sigma_a$

- Lemma 1: $L^a = \frac{1}{2}\eta^a_{\mu\nu} L_{\mu\nu}$

$= \frac{1}{2}\eta^a_{\mu\nu}(x_\mu p_\nu - x_\nu p_\mu)$

$= \frac{1}{2}\eta^a_{\mu\nu}(x_\mu p_\nu + \frac{1}{2}\eta^a_{\nu\mu} x_\nu p_\mu$

$= \eta^a_{\mu\nu} x_\mu p_\nu = \eta^a_{\mu\nu} x_\mu (-i\hbar) \frac{\partial}{\partial x_\nu}$

Lemma 2: $\eta^a_{\mu\nu}\,\eta^b_{\mu\sigma} = ?$

$$\eta^a_{\mu\nu}\,\eta^b_{\mu\sigma} = \delta^{ab}\,\delta_{\sigma\nu} + \varepsilon^{abc}\,\eta^c_{\nu\sigma}$$

$$\eta^a_{\mu\nu}\,\eta^b_{\mu\sigma}\,\frac{1}{4(1+x_5)^2}\,x_\nu x_\sigma\,\sigma_a\sigma_b$$

$$= \frac{1}{4(1+x_5)^2}\left(\sigma_a\cdot\sigma_a\;x_\nu x_\sigma\right) + \varepsilon^{abc}\,\cancel{x_\nu\,\eta^c_{\nu\sigma}\,x_\sigma\,\sigma_c)}$$

$$= \frac{1-x_5^2}{4(1+x_5)^2}\,\sigma_a\cdot\sigma_a$$

$$= \frac{1-x_5}{4(1+x_5)}\,\sigma_a\cdot\sigma_b$$

Lemma: $\dfrac{1-x_5}{4(1+x_5)}\,\sigma_a\cdot\sigma_a$

Four-dimensional Quantum spin-Hall

其中 $-i\hbar\frac{\partial}{\partial x_i}\cdot\frac{-eg/c}{2(1+x_5)}\eta^a_{im}x_m e_a$

$=i\hbar\frac{eg}{2c}\frac{1}{(1+x_5)}\eta^a_{im}x_m\frac{\partial}{\partial x_i}e_a$

$=-\frac{eg}{2c}\frac{1}{1+x_5}\vec{L}^a\cdot\vec{e_a}$

$K_a=\frac{1}{2}\eta^a_{\mu\nu}L_{\mu\nu}=\frac{1}{2}\eta_{\mu\nu}(x_\mu p_\nu-x_\nu p_\mu)$

$=\eta^a_{\mu\nu}x_\mu p_\nu$

- configuration $A_\mu=\frac{1}{\sqrt{2}}(\eta^a_{\mu\nu}+\bar{\eta}^a_{\mu\nu})x_\nu\sigma_a$

- Lemma 1: $L^a=\frac{1}{2}\eta^a_{\mu\nu}L_{\mu\nu}$

$=\frac{1}{2}\eta^a_{\mu\nu}(x_\mu p_\nu-x_\nu p_\mu)$

$=\frac{1}{2}\eta^a_{\mu\nu}(x_\mu p_\nu+\frac{1}{2}\eta^a_{\nu\mu}x_\nu p_\mu$

$=\eta^a_{\mu\nu}x_\mu p_\nu=\eta^a_{\mu\nu}x_\mu-i\hbar\frac{\partial}{\partial x_\nu}$

Lemma 2: $\eta^{a}_{\mu\nu} \eta^{b}_{\mu\sigma} = ?$

$$\eta^{a}_{\mu\nu} \eta^{b}_{\mu\sigma} = \delta^{ab} \delta_{\sigma\nu} + \epsilon^{abc} \eta^{c}_{\nu\sigma}$$

$$\eta^{a}_{\mu\nu} \eta^{b}_{\mu\sigma} \frac{1}{4(1+x_5)^2} x_\nu x_\sigma \sigma_a \sigma_b$$

$$= \frac{1}{4(1+x_5)^2} \left(\sigma_a \cdot \sigma_a \, x_\nu x_\sigma \right) + \epsilon^{abc} \cancel{x_\nu \eta^{c}_{\nu\sigma} x_\sigma \sigma_c)}$$

$$= \frac{1-x_5^2}{4(1+x_5)^2} \sigma_a \cdot \sigma_a$$

$$= \frac{1-x_5}{4(1+x_5)} \sigma_a \cdot \sigma_b$$

Lemma: $\frac{1-x_5}{4(1+x_5)} \sigma_a \cdot \sigma_a$

Four-dimensional Quantum spin-Hall

$$H = -\frac{\hbar^2}{2m}\left(\nabla^2 - \frac{1-x_5}{4(1+x_5)}\,\sigma_a\cdot\sigma_a - \frac{\cancel{1-x_5}\ 1}{1+x_5}\,L_a\cdot\sigma_a\right)$$

$$\frac{\sin^2\frac{\theta}{2}}{4\cdot 2\cdot\cos^2\frac{\theta}{2}}\,\sigma_a\cdot\sigma_a + \frac{4}{2\cos^2\frac{\theta}{2}}\,L_a\cdot\sigma_a$$

其中： $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^4\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta\frac{\partial}{\partial\theta}\right)$

$$-\frac{4}{r^2\sin^2\theta}\left(2\,\cancel{L^2+2K^2}\ 4L^2\right)$$

其中K与L是一样的

$$\therefore H = -\frac{\hbar^2}{2m}\left\{\frac{1}{r^2\sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta\frac{\partial}{\partial\theta}\right) - \right.$$

$$\frac{2(L^2+K^2)}{\sin\theta^2} + \frac{(1-\cos\theta)}{\cancel{4\cos^2\frac{\theta}{2}}\ \sin^2\theta}\left[J_a^2 - L^2 - \sigma_a^2\right]/2$$

$2(1+\cos\theta)$

$$4L^2 - (1-\cos\theta) = (3+\cos\theta)L^2$$

$$\Rightarrow A = \frac{1}{\sin\theta^3}\frac{\partial}{\partial\theta}\sin^3\theta\frac{\partial}{\partial\theta} - \frac{1-\cos\theta}{(1+\cos\theta)^2}\,\sigma_a\cdot\sigma_a$$

$$-\frac{\cancel{1}\ (1-\cos\theta)}{\frac{(1-\cos\theta)}{\cancel{2\sin^2\frac{\theta}{2}}\ \sin^2\theta}}\left(J^2 - L^2 - I^2\right) - \frac{4L^2}{\sin^2\theta}$$

解: $\frac{1}{1+x_5} L_a \cdot T_a = \frac{1-\cos\theta}{\sin^2\theta} \underline{L_a \cdot T_a}$

$\frac{1}{\sin\theta^2} L^2 = \qquad = \frac{1-\cos\theta}{\sin^2\theta}\left(\frac{J_a^2 - L_a^2 - T_a^2}{2}\right)$

$-4 + \underline{2(1-\cos\theta)} = -2(1+\cos\theta)$

$\frac{1-x_3}{1+x_5} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$T_a \cdot - \frac{J_a^2}{1+x_5} - \frac{L_a^2}{1+x_5} - \frac{x_5}{1+x_5} T_a \cdot T_a$

$\Downarrow$

$\frac{4L_a^2}{\sin\theta^2} \qquad \frac{2(1-\cos\theta)}{\sin^2\theta} J_a^2 - \frac{2(1-\cos\theta)}{\sin^2\theta} L_a^2$

$\frac{2(1-\cos\theta)\cdot\cos\theta}{1\sin^2\theta} \underline{T_a \cdot T_a}$

$2(1-\cos\theta) - (1-\cos\theta)(3+\cos\theta)$

$= -(1-\cos\theta)(1+\cos\theta)$

Lemma 3: $\frac{1}{1+x_5} = \frac{1}{2\cos^2\frac{\theta}{2}} = \frac{1-\cos\theta}{\sin^2\theta}$

$$\left(\frac{1-x_5}{1+x_5}\right)\alpha^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}\alpha^2(\sigma_\alpha\sigma_\alpha/4)$$

$$2\frac{\cancel{1-x_5}\,1}{1+x_5}\alpha\, L_\alpha\cdot\sigma_\alpha = \frac{1-\cos\theta}{\sin^2\theta}\left(\frac{J_\alpha^2 - L_\alpha^2 - \sigma_\alpha^2}{\cancel{2}}\right)\underline{(\alpha/2)}$$

$\Downarrow$

$\underline{\text{Let}}: \alpha = \frac{1}{2}$

$$(1-\cos\theta)(1-\cos\theta-2)$$

$\frac{\alpha}{2} = 2\alpha^{\cancel{2}}$

$$-i\hbar\frac{\partial}{\partial x_i} - \frac{e}{c}A_i$$

$\Downarrow$

$\alpha = \frac{1}{4}$

$$2 - i\hbar\frac{\partial}{\partial x_i}\cdot\eta^a_{im}x_m\sigma_a/2 \quad \underline{L_{uv}} \quad 4\left(L_\alpha\cdot\frac{\sigma_\alpha}{2}\right)$$

即: $\frac{(1-\cos\theta)^2}{\sin^2\theta}S_\alpha\cdot S_\alpha + \frac{2(1-\cos\theta)}{\sin^2\theta}(J_\alpha^2 - L_\alpha^2 - \sigma_\alpha^2)$

$(1/2)$ 为 $\frac{1}{2}\sigma_\alpha$

$$\frac{1}{2}\eta^a_{uv}(x_u p_v - x_v p_u) = 2\eta^a_{uv}L_{uv}\cdot S_\alpha$$

$$= \frac{2(1-\cos\theta)}{\sin^2\theta}J_\alpha^2 - \frac{2(1-\cos\theta)}{\sin^2\theta}L_\alpha^2 - \vec{S}_\alpha\cdot\vec{S}_\alpha$$

问题: 为什么是 $2(J_\alpha^2 - L_\alpha^2 - S_\alpha^2)$

$$= 4\,L_\alpha\cdot S_\alpha = 2L_\alpha\cdot\sigma_\alpha$$

因为：$\boxed{K_\alpha = \frac{1}{4}\eta^{a}_{uv} L_{uv}}$ 所以量相化

从而：$\Delta = \frac{1}{r^4}\frac{\partial}{\partial r}\left(r^4\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta\frac{\partial}{\partial\theta}\right)$

$- \frac{1}{\sin^2\theta}\left(2L^2 + 2K^2\right)$

$- \frac{(1-\cos\theta)^2}{\sin^2\theta}\hat{S}_\alpha \cdot \hat{S}_\alpha - \frac{2(1-\cos\theta)}{\sin^2\theta}\left(J_\alpha^2 - L_\alpha^2 - S_\alpha^2\right)$

$= \frac{1}{r^4}\frac{\partial}{\partial r}\left(r^4\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta\frac{\partial}{\partial\theta}\right)$

$- \frac{1}{\sin^2\theta}\left[2(1-\cos\theta)J_\alpha^2 + 2(1+\cos\theta)L_\alpha^2 \cancel{+} S_\alpha^2\right]$

即 $f(\theta,\alpha,\beta,\gamma) = d(\theta)D(\alpha,\beta,\gamma)$

$\otimes D(\alpha_I,\beta_I,\gamma_I)$

$\frac{1}{\sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\left[\left(\overbrace{2(j+1)j - 2k(k+1)}^{(2j+2k+1)(j-k)}\right)\right.$

$\left.\cos\theta + 2j(j+1) + 2(k(k+1)) - l(l+1)\right]$

$\frac{1}{\sin\theta}\left(\frac{\partial}{\sin\theta}\frac{\partial}{\partial\theta}P(\theta)\right) - \quad (j-k) \quad -(j+k+1)\cos\theta$

$\Uparrow$

$\underline{2j(j+1) + 2k(k+1)} \; -2(j-k)(j+k+1)\cos\theta$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{\left((j-k)-(j+k+1)\cos\theta\right)^2}{\sin^2\theta}$$

$$-(j+k+1)^2 - 2 + I(I+1) \equiv -\lambda$$ (monopole harmonics)

$$\Rightarrow \left(-\lambda - 2 + (j+k+1)^2 - I(I+1)\right) \leq \boxed{j+k+1}$$

其中 $\lambda \equiv \frac{p^2+q^2}{2} + 2p + q$; 考虑 leading state:

$p=2j$; $q=2k$: $\lambda = 2j^2+2k^2+4j+2k$

$$\cancel{2j^2} + \cancel{2k^2} + 4j + 2k + 2 - j^2 - k^2 - 1 - \cancel{2jk} - 2j - 2k$$

$$= (j-k)^2 + 2j + 1 - I(I+1) + j - k - 1$$

$(j-k)^2$ $\qquad (j-k)(j-k+1)$

$(j+k)^2$ $\qquad j+(j+1)$

$$(j-k)(j-k+1) \geq I(I+1)$$

由于 $j-k \leq I \Rightarrow (j-k \equiv I)$

其中 I 可以取 half integer

• 其中：$\alpha = \pm 2j+1$ $\beta = \mp(2j+1)$

例：~~α~~ $D^{2j+1}_{2j+1,2l+1}(\theta)$ ~~D~~

即：SU(2) harmonics:

⇓

$\boxed{\text{即:}}$ $X_{I,n;j_1,m_1;j_2,m_2} = s_{\ell}\, h\bar{o}^{\prime}\, d^{(2j+1)}_{0,2j+1}(\theta)\; U\ldots$

⇓

这个即为 SO(5) harmonic 的 CG coefficients：

我直接利用 C.N.Yang 的波函数去计算 CG 系数

• 但是 C.N.Y 并没给出这个 $X_{j,m_1;j_2,m_2}$ 是什么？

conformal theory

1. 先把 Monopole Harmonics 搞明白。

2. 研究 4D Quantum spin Quantum Hall

将会很简单

3. C.N.Y 这个文章基本搞明白了。

$\boxed{\text{计算 SO(5) CG 系数}}$